IV.—*On a method of comparing the light of the sun with that of the fixed stars.* *By* William Hyde Wollaston, *M.D. F.R.S.*

Read December 11, 1828.

One of the most ingenious contributors to the Transactions of our Society in the last century, the Rev. John Michell, in a paper intituled "An inquiry into the probable parallax and magnitude of fixed stars, &c*." has proposed it to astronomers, as an object worthy their attention, to determine what proportion the light, afforded us separately by each fixed star, bears to the light which we receive from the sun; since, from our inability to measure the annual parallax of those very remote bodies, such a comparison is the best, perhaps the only method within our reach, of obtaining, though not certain, yet probable estimates of their distances; and thus forming reasonable conjectures concerning the extent of the visible universe. In order that we may judge, with the least chance of error, of the mean distance of those stars which are the nearest to the earth, he directs us to compare the light of the brightest stars with that of the sun, and next to calculate how far the sun must be removed, to make the light that we should then receive from him, not more than equal to the mean light of the stars chosen for comparison.

Mr. Michell made, as he says, some rude experiments for determining the comparative brilliancy of certain principal stars; but has not suggested any contrivance for comparing a star with the sun. He states, however, so distinctly the great object of such a comparison, and the inferences which an industrious observer would thence be entitled to draw, concerning the distances of those stars whose light he might succeed in measuring, that it is surprising that no astronomer has been incited by these remarks to devise a method of making the requisite observations, and that now, so many years after Mr. Michell's suggestion was made public, so much remains to be effected in this branch of photometry.

* Phil. Trans. 1767: p. 234.

From a comparison which I made in the year 1799 (by a method described in the note subjoined) of the light of the sun with that of the moon, I should estimate the direct light of the sun as being nearly one million times greater than that of the moon*; and consequently the direct light of the sun as very many millions times greater than that afforded us by all the fixed stars, taken collectively. Such then being, to our visual organs, the vast disproportion in radiance between the sun and the whole starry firmament, it is not to be expected that we should assign very accurately how much greater the light of

* The observations on which this estimate is founded, are given in detail at the end of the Appendix to this paper. The mode of making the observations was the following.

The sun's light was compared with that of a candle, by admitting a beam of it into a room through a small circular hole in a plate of metal, fastened in a window-shutter; and a small cylinder of any opaque material being placed in the beam, so as to cast a shadow upon a screen, the distance of a candle from the same cylinder (or an equal one placed at the same distance from the screen) was varied, until the shadow in the line of the candle became equally intense with the shadow in the line of the sun. The direct light of the moon was compared with the light of a candle in the same manner. This method of comparing lights by the intensity of the shadows which they occasion, was pursued also by Count RUMFORD.

It appears from the mean of the observations given in No. V. of the Appendix, that the light of the sun is equal to that of 5563 candles placed at the distance of one foot; a result which accords very nearly with that of BOUGUER. For he states the light of the sun to be equal to that of 11,664 wax candles at the distance of 16 inches French, which is equivalent to 5774 wax candles at the distance of one foot English. It appears also from my experiments, that the light of the full moon is equal to $\frac{1}{144}$th part of the light of a candle, placed at the distance of a foot; and hence, that the sun's light is equal to $5563 \times 144 \times$ moon's light $= 801{,}072. \times$ moon's light. BOUGUER, who differs greatly from me in the comparison of the moon with a candle, states the light of the sun to be $= 300{,}000 \times$ moon's light. The proportion which the light of the full moon ought to bear to the light of the sun, on the supposition that the moon gives off again all the solar light that falls upon it, has been differently estimated by several mathematicians who have computed it. The light of the sun at the earth being represented by unity, Mr. MICHELL expresses that of the full moon by $\sin^2 \frac{1}{2}$ ☾ diameter $= \frac{1}{450{,}000}$. EULER, in the Transactions of the Berlin Academy for 1750, represents the light of the full moon by $\frac{1}{2} \sin^2 \frac{1}{4}$ ☾ diameter, which is only $\frac{1}{8}$th of the former expression of Mr. MICHELL. Neither of these expressions, however, appears to be correct. For if we consider that the quantity of solar light which falls upon any point in the moon's surface, must vary, if we regard the sun's rays as parallel, as the cosine of the angular distance of that point from the point in the moon over which the sun is vertical, we shall obtain, by following EULER's own method, the formula $\frac{1 + 2 \sin^3 \text{☾ semidiameter} - \cos^3 \text{☾ semidiameter}}{3}$, to express the quantity of light, which, on the given supposition, we ought to receive from the moon; and this expression reduced to numbers $= \frac{1}{100{,}000}$. The moon therefore appears to give off only about $\frac{1}{8}$th of the light which she receives.

the sun is, than that exceedingly minute quantity of it which shines upon us from any one, even the most brilliant of the fixed stars.

It may be remembered that on a former occasion, in examining the correct performance of a good telescope, I found that the sun's image, reflected from the surface of a small sphere, (such as that of a thermometer-bulb filled with mercury,) and viewed at a proper distance through a telescope, is, to appearance, extremely like a fixed star, and forms, in such experiments, an admirable substitute for one, in being really fixed, and therefore well adapted for deliberate observation. It occurred to me, while engaged in this examination, that by comparing such a reflected image with one of the larger stars, I might be able to obtain some grounds for estimating the light of the star.

It would be desireable, though extremely difficult, in conducting such an experiment, to make a direct comparison between the star and the sun's image; since in that case we should be enabled to avoid the uncertainties inseparable from an indirect comparison, the consequence of observing at times so distant, that the atmosphere in the interval has undergone considerable change. As, however, the only practicable method of observing is the indirect one, by comparing the two objects with some common standard at different times, we must endeavour to remove those uncertainties from our results, by repeating each series of comparisons so frequently, that the average of each series may be affected by atmospheric vicissitudes, or may fairly be presumed to be so, in an equal degree.

The common standard of comparison which I chose, was the image of a candle, reflected from a small thermometer-bulb, (in most trials about $\frac{1}{4}$th of an inch in diameter,) filled with mercury, and seen by one eye through a lens of about two inches focus, at the same time that the sun's image, reflected (in the manner above described) from a thermometer-bulb placed at a distance, or the star itself, was viewed by the other eye through a telescope.

In order to make the light of the two objects, when seen through the telescope, and that of the candle, more nearly alike in colour, I placed two yellow glasses at the eye-piece; and I thought it expedient to have in view, at the same time with the subject of comparison, two candles, one of tallow, the other of wax; that by making the star, or the little sun, a mean between the two lights, I might obtain a nearer approximation to the truth*. The measure taken in each experiment was the distance of the two candles from the bulb; and every distance that I have reported amongst the observations, was the mean result of several trials.

In reducing these observations we have to consider that though the image of the sun, which is half the radius distant from the centre of the bulb, subtends at its surface the same angle, of half a degree, as the sun itself, and therefore to an eye placed at the surface would appear equally brilliant with the sun itself; yet the apparent diameter of this little sun will decrease in proportion as the eye recedes from the bulb, so that at the distance of D inches, the apparent diameter of the image will be reduced in the ratio of $\frac{1}{4}$th of the diameter of the bulb, or of $\frac{B}{4}$, to D, and consequently the brightness of the image will be reduced in the ratio of 1 to the square of $\frac{4\,D}{B}$.

If the distance of the eye from the bulb be so chosen, that, on comparing the little sun and the star, separately, with the candle's image, the candle in the two cases is at unequal distances from its bulb, d being made to represent the candle's distance from the bulb in comparing it with the sun, and δ the candle's distance from the bulb in comparing it with the star, $\frac{4\,D}{B} \times \frac{\delta}{d}$ will be the distance at which the little sun would appear of equal brightness with the star, and the brightness of the little sun would then be to the brightness of the sun itself as 1 to $\left[\frac{4\,D \times \delta}{B \times d}\right]^2$.

If, in two comparisons made, the one between the candle and the sun, the other between the candle and a star, the candle be reflected by bulbs of different diameters, and viewed with lenses of unequal focal length, the apparent diameter of the candle's image will be as the diameter of the bulb directly, and as the focal length of the lens inversely; and hence, if b be the diameter of the bulb and l the focal length of the lens in comparing the candle with the sun, and β be the diameter of the bulb and λ the focal length of the lens in com-

* If any other artificial light could be found, which would at all times be of uniform brilliancy, and of so white a colour as to supersede the necessity of using yellow glasses, it would of course be preferable, as a standard, to the light of a candle.

paring the candle with the star, $\frac{4\,D}{B} \times \frac{\delta}{d} \times \frac{\lambda}{l} \times \frac{b}{\beta}$ will be the distance at which the little sun would appear of equal brightness with the star; and the brightness of the little sun would then be to the brightness of the sun itself as 1 to $\left[\frac{4\,D \times \delta \times \lambda \times b}{B \times d \times l \times \beta}\right]^2$; and it is according to the latter formula that the observations, made with bulbs of different diameter and with lenses of different focal length, have been reduced so as to be compared in No IV. of the Appendix.

The first star that I compared with the sun, was Sirius; and the observations were made at times when, the altitudes of the two bodies being not very widely different, their powers of illumination might be presumed to be affected, on the average, in almost an equal degree by the atmosphere. The table of reduced observations [No. IV. of the Appendix], in which each of seven observations of the sun is compared with each of seven observations of Sirius, will be found to exhibit discordances, which are referrible, probably, to our variable climate, and to the smoky atmosphere of London. Uniformly transparent skies are requisite to give uniformity to such experiments; and in our climate, therefore, though the mean of very many comparisons would, probably, give a result not very remote from the mean of a much smaller number of trials made under a less variable atmosphere, we must expect the greatest and least results to differ widely from one another*.

The mean of the various trials seems to show, that the light of Sirius is equal to that of the sun reflected from the surface of a sphere $\frac{1}{10}$th of an inch in diameter, and seen at the distance of about 210 feet. The diameter of such an image of the sun, is to that of the sun itself as 1 to 100,000; and, consequently, the brightness of the image would be to the brightness of the sun itself as 1 to 10,000,000,000; but as nearly half of the light must be lost during reflection, we are not warranted by these experiments in supposing that the light of Sirius exceeds a 20,000,000,000th part of the sun's light.

* An observer, intending to pursue this inquiry, would do well, therefore, to choose a favourable climate; and, further, he ought to select such stars for comparing with the sun, as have, severally, at the times of observation, nearly the same altitude with the sun. The accuracy of these comparisons with the sun would admit of rigorous investigation, by comparing the same stars with one another. Stars having the same Æ might be compared at places having different latitudes, or even in different hemispheres, whereby the unequal influence of the atmosphere at different altitudes might be wholly eliminated.

Were the sun removed to such a distance, that the light which we received from it were only a twenty thousand millionth part of its present light, that distance would be equal to $\sqrt{2} \times 100{,}000 \times$ its present distance, and it would, if still situated in the ecliptic, have a parallax in longitude of nearly $3''$; but if placed at the same angular distance from the ecliptic, as Sirius, since the parallax varies as the sine of a star's latitude, and the latitude of Sirius is about $39^\circ\frac{1}{2}$, it would have a parallax in latitude of about $1''\frac{8}{10}$.

Assuming the parallax of Sirius to be half a second, and consequently its distance from the earth to be 525,481 times the distance of the sun from the earth, Sirius, if placed at the sun's distance, would subtend 3.7 times the sun's apparent diameter, and would afford us as much light as 13.8 suns.

From similar experiments to those I made on Sirius, it appeared that the light afforded us by Lyra was about $\frac{1}{180{,}000{,}000{,}000}$th part of the sun's light, or about $\frac{1}{9}$th part of the light of Sirius.

Without extending this method to a comparison of the stars with the sun, we may confine it, if we think proper, to comparing the stars with one another, so that, in fulfilment of the wishes of Mr. MICHELL*, "instead of distributing them, as has hitherto been done, into a few ill-defined classes, they may be ranked with precision, both according to their respective brightness, and the exact degree of it."

In concluding the paper which is now submitted to the Society, I request them to direct their attention rather to the method than to the observations; for these have been much too few in number to enable me to state with any degree of confidence, what proportion the light of the sun really does bear to either of the stars compared with it. It was my intention, had my health permitted it, to have proceeded with this inquiry, until by multiplied observations I had ascertained how nearly the mean of one extensive series of comparisons accorded with the mean of another series; and how far, therefore, the method itself was deserving of confidence. But since I have now no prospect of bringing the subject to perfection, I submit the method itself to the consideration of industrious observers, who will soon be able to judge of the expediency of continuing the inquiry.

* Phil. Trans. 1767: p. 241.

Appendix.

I. Observations of a reflected image of the Sun, compared with a similar image of a Candle.

The sun's image, reflected from a thermometer-bulb filled with mercury, is viewed from a distance by the observer through a telescope, with power 36, and two yellow glasses at the eye-piece. The image of a candle, reflected from a similar bulb, is viewed through a lens of from 2 to 2½ inches focus; and the distance of the candle is varied, until its image appears of equal brightness with the image of the sun.

Date of the Observation.	B Diameter of the thermometer-bulb, for ⊙, in inches.	D Distance of the thermometer-bulb, for ⊙, in inches.	*b* Diameter of the thermometer-bulb, for Candle, in inches.	*d* Distance of the thermometer-bulb, from the Candle, in inches.	*l* Focus of Lens for viewing the image of Candle, in inches.
1826, March 10	.19	1440	.44	68	2.0
1827, March 14	.26	2928	.26	42	2.5
.... March 16	.26	2928	.26	28	2.5
.... March 16*	.11	1440	.26	41	2.5
.... March 25	.26	2928	.26	36	2.5
.... March 25*	.11	1440	.26	57	2.5
.... April. 6	.11	1440	.26	49	2.5

II. Observations of Sirius, compared with a reflected image of a Candle.

Sirius is viewed by the observer through a telescope, with power 36, and two yellow glasses at the eye-piece. The image of a candle, reflected from a thermometer-bulb filled with mercury, is viewed through a lens of from 2 to 2½ inches focus; and the distance of the candle is varied, until its image appears of equal brightness with that of Sirius.

Date of the Observation.	β Diameter of the Thermometer-bulb, for Candle, in inches.	δ Distance of the Thermometer-bulb, from the Candle, in inches.	λ Focus of Lens for viewing the image of the Candle, in inches.	Remarks.
1826, March 15	.44	216	2.0	
.... March 19	.44	165	2.0	Very bright night.
1827, Feb^y. 14	.44	246	2.0	
.... Feb^y. 15	.44	170	2.0	Very bright night, $10^h\ 30^m$.
.... March 14	.26	102	2.5	
.... April. 4	.26	90	2.5	
.... April. 9	.26	93	2.5	$7^h\ 15^m$.

III. Observations of α Lyræ, compared with a reflected image of a Candle.

1827, April 9	.26	276	2.5	

IV. Reduction of the preceding observations of the Sun and Sirius.

If all the light of the sun, which falls on the thermometer-bulb, be reflected by it, the light of $\odot = \left[\frac{4\,D \times b \times \delta \times \lambda}{B \times \beta \times d \times l}\right]^2 \times$ the light of Sirius; and since there are seven observations of the sun compared with a candle, and seven of Sirius, there will be forty-nine different values of the expression $\frac{4\,D \times b \times \delta \times \lambda}{B \times \beta \times d \times l}$; which are all inserted in the following Table.

Observations of ⊙.	1826. March 10.	1827. March 2.	1827. March 16.	1827. March 16*.	1827. March 25.	1827. March 25*.	1827. April 6.	Totals.
Of Sirius.								
1826, March 15	96.297	107.022	160.533	127.441	124.859	91.668	106.634	814.454
.... March 19	73.560	81.732	122.629	97.351	95.398	70.024	81.458	622.152
1827, Feb^y 14	109.672	121.886	182.829	145.141	142.200	104.400	121.725	927.853
.... Feb^y 15	75.789	84.230	126.345	100.301	98.268	72.146	83.925	641.004
.... March 14	98.435	109.397	164.096	127.630	130.270	93.703	109.002	832.533
.... April 4	86.854	96.527	144.791	112.625	114.944	82.679	96.178	734.598
.... April 9	89.749	99.745	149.622	116.366	118.776	85.435	99.384	759.077
								5.331.671

$$\frac{5.331.671}{49} = 108.809$$

Hence the mean result of the foregoing experiments is that, supposing none of the Sun's light to be lost on reflection at the thermometer-bulb,

$$\odot\text{'s light} = 108.809^2 \times \text{light of Sirius}$$

$$= 11.839.533.000 \times \text{the light of Sirius};$$

but, allowing for the loss of nearly half the light on reflection, that

$$\odot\text{'s light} = 20.000.000.000 \times \text{the light of Sirius}.$$

V. Observations of the Light of the Sun, compared with that of a Candle, by means of Shadows.

Date of the Observation. 1799.	H Diameter of the Hole in the Shutter, in parts of an inch.	D Distance of the Hole from the Screen in inches.	C Distance of the Candle from the Screen in inches when its light is equal to that of ☉, admitted through Hole.	Numerical value of the Expression $\left[\frac{12 \times D}{C \times H} \times 2 \tan ☉\text{'s} \frac{\text{Diam.}}{2}\right]^2$
End of May,	.0067	93	19.5	6152
and	.0072	93	19.0	5611
Beginning of June.	.0086	93	18.0	4382
	.0093	93	17.5	3965
	.0093	111.5	20.5	5228
	.0098	102	14.25	6477
	.0098	108	$15.\frac{1}{6}$	6410
	.0098	120	17.0	6299
May 28	.0098	120	17.5	5944
June 19	.0105	126	15.0	7770
	.0111	93	14.5	4054
	.0118	93	13.0	4463
				66755

$$\frac{66755}{12} = 5563$$

Hence 5563 is the number of Candles, which being placed at the distance of twelve inches, will give a light equal to that of the Sun.

VI. Observations of the light of the Moon compared with that of a Candle, by means of Shadows.

Date of the Observation.	Remarks.	Distance, in inches, of the Candle from a Screen, when its light is equal to that of ☾.
1799, May 16	☾'s Elongation 170°½	144
.... June 17	☾ Full	144

Hence, $☾ = \frac{1}{144} \times$ Candle placed at the distance of twelve feet,

and $☉ = 5563 \times \left(\frac{144}{12}\right)^2$ Moons.

$= 801.072$ Moons.